

RELATIVISTIC ELECTRON BEAMS

Here $\gamma = (1 - \beta^2)^{-1/2}$ is the relativistic scaling factor; quantities in analytic formulas are expressed in SI or cgs units, as indicated; in numerical formulas, I is in amperes (A), B is in gauss (G), electron linear density N is in cm^{-1} , and temperature, voltage and energy are in MeV; $\beta_z = v_z/c$; k is Boltzmann's constant.

Relativistic electron gyroradius:

$$r_e = \frac{mc^2}{eB}(\gamma^2 - 1)^{1/2} \text{ (cgs)} = 1.70 \times 10^3(\gamma^2 - 1)^{1/2} B^{-1} \text{ cm.}$$

Relativistic electron energy:

$$W = mc^2\gamma = 0.511\gamma \text{ MeV.}$$

Bennett pinch condition:

$$I^2 = 2Nk(T_e + T_i)c^2 \text{ (cgs)} = 3.20 \times 10^{-4}N(T_e + T_i) \text{ A}^2.$$

Alfvén-Lawson limit:

$$I_A = (mc^3/e)\beta_z\gamma \text{ (cgs)} = (4\pi mc/\mu_0 e)\beta_z\gamma \text{ (SI)} = 1.70 \times 10^4\beta_z\gamma \text{ A.}$$

The ratio of net current to I_A is

$$\frac{I}{I_A} = \frac{\nu}{\gamma}.$$

Here $\nu = Nr_e$ is the Budker number, where $r_e = e^2/mc^2 = 2.82 \times 10^{-13} \text{ cm}$ is the classical electron radius. Beam electron number density is

$$n_b = 2.08 \times 10^8 J\beta^{-1} \text{ cm}^{-3},$$

where J is the current density in A cm^{-2} . For a uniform beam of radius a (in cm),

$$n_b = 6.63 \times 10^7 I a^{-2} \beta^{-1} \text{ cm}^{-3},$$

and

$$\frac{2r_e}{a} = \frac{\nu}{\gamma}.$$

Child's law: (non-relativistic) space-charge-limited current density between parallel plates with voltage drop V (in MV) and separation d (in cm) is

$$J = 2.34 \times 10^3 V^{3/2} d^{-2} \text{ A cm}^{-2}.$$

The saturated parapotential current (magnetically self-limited flow along equipotentials in pinched diodes and transmission lines) is²⁹

$$I_p = 8.5 \times 10^3 G \gamma \ln \left[\gamma + (\gamma^2 - 1)^{1/2} \right] \text{ A},$$

where G is a geometrical factor depending on the diode structure:

$$\begin{aligned} G &= \frac{w}{2\pi d} && \text{for parallel plane cathode and anode} \\ &&& \text{of width } w, \text{ separation } d; \\ G &= \left(\ln \frac{R_2}{R_1} \right)^{-1} && \text{for cylinders of radii } R_1 \text{ (inner) and } R_2 \text{ (outer);} \\ G &= \frac{R_c}{d_0} && \text{for conical cathode of radius } R_c, \text{ maximum} \\ &&& \text{separation } d_0 \text{ (at } r = R_c \text{) from plane anode.} \end{aligned}$$

For $\beta \rightarrow 0$ ($\gamma \rightarrow 1$), both I_A and I_p vanish.

The condition for a longitudinal magnetic field B_z to suppress filamentation in a beam of current density J (in A cm^{-2}) is

$$B_z > 47\beta_z(\gamma J)^{1/2} \text{ G}.$$

Voltage registered by Rogowski coil of minor cross-sectional area A , n turns, major radius a , inductance L , external resistance R and capacitance C (all in SI):

$$\begin{aligned} \text{externally integrated} & \quad V = (1/RC)(nA\mu_0 I/2\pi a); \\ \text{self-integrating} & \quad V = (R/L)(nA\mu_0 I/2\pi a) = RI/n. \end{aligned}$$

X-ray production, target with average atomic number Z ($V \lesssim 5 \text{ MeV}$):

$$\eta \equiv \text{x-ray power/beam power} = 7 \times 10^{-4} ZV.$$

X-ray dose at 1 meter generated by an e-beam depositing total charge Q coulombs while $V \geq 0.84V_{\max}$ in material with charge state Z :

$$D = 150V_{\max}^{2.8} QZ^{1/2} \text{ rads}.$$